Meandering of Water Rivulets

Water rivulets are produced on inclined flat plates made of high-density polyethylene, Mylar, Plexiglas and acetate film. Conditions are obtained for straight rivulets to begin to meander and for measuring rivulets to cease their unsteadiness. These critical conditions are correlated using a one-dimensional theory of high Reynolds number slender rivulets in which pressure gradients generated by curvature, surface tension and contact-angle hysteresis balance.

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SCOPE

Rivulets are slender filaments of liquid flowing along inclined solid surfaces. At sufficiently high flow rates, such flows undergo an instability leading to sinuous structured flows called meanders. In this study, the conditions needed to produce, and to sustain such flows are examined. Water rivulets are formed on solid plates of high density polyethylene, plexiglas, mylar, and acetate film. Critical values of flow rate and plate inclination are experimentally determined under carefully controlled conditions. A correlation of the data against a single dimensionless parameter yields insight to the underlying mechanism for the instability.

CONCLUSIONS AND SIGNIFICANCE

The results for water rivulets show that the meandering properties are repeatable only if the history of formation of the rivulets is controlled. In this case Q^- and Q^+ can be well-correlated using a stability index I containing pressure gradients, surface tension and contact-angle hysteresis. The results thus give for high Reynolds number rivulets a predictable measure of rivulet properties that would be useful in accessing the heat/mass transport across such flows.

INTRODUCTION

A rivulet is a narrow stream of liquid flowing down a solid surface. The rivulet possesses a curved interface with the surrounding gas; this interface intersects the solid in a pair of contact lines. The component of gravity down the solid drives the flow while the component normal to the solid, the surface tension and contact angle determine the shape of the liquid-gas interface.

There exists a *straight-rivulet* solution, Towell and Rothfeld (1966), which consists of a cylindrical static meniscus, straight, parallel static contact lines and unidirectional parallel flow down the solid. The straight-rivulet flow is susceptible to various instabilities (Kern, 1969, 1971; Culkin, 1979; Davis, 1980; Culkin, 1981; Weiland and Davis, 1981). The hydrodynamic state of the rivulet can greatly affect the heat/mass transport rates across rivulets in process equipment and so one wishes to know the stability characteristics of rivulets. Furthermore, rivulet instabilities often involve the movement of the contact lines. Hence, detailed understandings of rivulet instabilities can shed light on the mechanics of contact lines moving under unsteady conditions. Such knowledge is important in processes of spreading, coating and dry-patch formation.

In the present study we focus on the meandering of rivulets. A meander is an instability of the straight-rivulet flow resulting in a (steady or unsteady) sinuous deviation of the flow axis and contact lines. In the steady case, the meandering rivulet wets a fixed part of the solid surface while in the unsteady case portions of the solid are successively wetted and dewetted. Rivulet meanders were first documented by Kern (1969, 1971).

We have conducted a detailed experimental investigation into rivulet meandering. In these experiments distilled water exits a feed tube onto an inclined flat plate. We control the water flow rate and plate inclination. By varying the solid materials we examine the critical conditions necessary to produce a meander and necessary to sustain an already meandering rivulet. The systems examined involve water on substrates of Mylar, high-density polyethylene (HDPE), Plexiglas and acetate film. Thus, a variety of equilibrium contact angles as well as a range of contact-angle hysteresis are included in the study. An optical technique is used to determine advancing and receding contact angles, respectively, θ_A and θ_R. A simple one-dimensional theory for the meandering of high Reynolds number slender rivulets is presented and used to correlate the data taken. Such a theory allows us to obtain trends for the critical conditions as functions of the parameters of the problem.

MEANDER DEFINITION

The advancing and receding contact angles θ_A and θ_R are important in any consideration of rivulet dynamics since any rivulet configuration is in part determined by the contact-line conditions. If the hysteresis is appreciable, many possible values of contact angle can be assigned to fix the static equilibrium configuration of the straight rivulet. A great variety of straight rivulets is then

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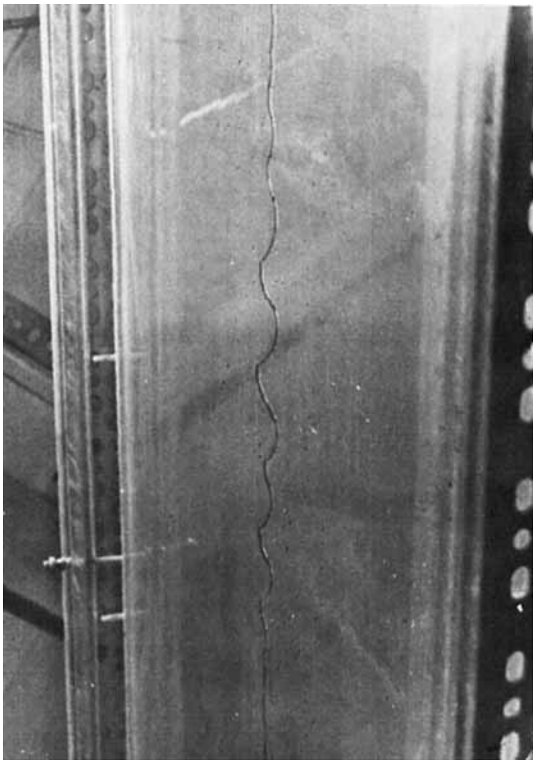


Figure 1. Meandering water rivulet.

possible. Moreover, any rivulet obtained is then dependent upon the history of formation of that particular rivulet.

In order to form a more concrete definition of meandering, consider the following thought study. We start with a straight rivulet; the contact lines are perfectly straight, the free surface is glassy smooth showing no waves or other unstetady behavior. We slowly increase the flow rate of the liquid. Initially, the rivulet height and width increase slightly in response to the greater discharge. Eventually, a critical flow rate is reached and meandering begins. The axis of the rivulet is now modulated. A typical meander of water upon plexiglas is shown in Figure 1. The spatial modulations grow in time as the meander evolves. This nascent meander can evolve to several different states.

For fixed supercritical flow rate the nascent meander can grow in amplitude until a steady meander is produced. In the steady meander all contact-line motion has ceased and the axis of the flow is now stationary but modulated in space. Alternatively, the nascent meander can evolve into a large amplitude state in which the rivulet axis has deviations which continually change in time. We thus have both stationary and unsteady meanders possible.

We find that the *history of formation* is crucial in leading to reproducible meander evolution. If the history of formation of the straight rivulet is carefully repeated, then the critical flow rate above which meandering occurs can be reproduced within about plus or minus 10%. Consider a newly formed water rivulet on a vertical HDPE plate. The flow rate needed to form a straight rivulet is much greater than that needed to sustain the rivulet against dropwise breakup to capillary instabilities. Having formed a straight rivulet as before, we now reduce the flow rate slowly. The rivulet cross-section becomes very small. The deviations in the contact-line position at points along the rivulet are as much as 50% of the average width of the rivulet. The flow is now highly corrugated, poor approximation to the idealized basic state straight rivulet.

If we begin increasing the flow rate, the critical flow rate at which meandering begins will be much lower than before. We are testing the stability of a different basic state and it is not surprising that the conditions for instability are also different. We, therefore, control not only the immediate circumstances pursuant to meandering, but the history of formation of the basic state as well.

We adopt a *formation protocol* for the establishment of straight rivulets; this will allow consistent study of conditions necessary to produce meandering. To form the straight rivulet, we increase the flow rate from zero until the dropwise flow merges to form a straight rivulet. We then increase the flow rate slowly until a nascent meandering appears. This process takes about one or two minutes. The rate at which the flow rate is increased is important. Sudden increases in flow rate can induce meandering at non-systematic flow rates substantially below that resulting from slow careful increases of flow rate.

For rivulets formed by the procedure described that flow rate at which the contact lines "break out" into a continuous evolution away from the straight rivulet we call *Q-from-below;* we denote this by Q^-. We do not distinguish between positions along the rivulet at which instability first appears. meandering usually begins far from the inlet. The instability then moves progressively upward toward the nozzle as time increases. For water on HDPE, the unsteady meandering instability can eventually encompass all but a short entrance region below the nozzle and so Q^- is a valid experimental measure. We find no correlation between distance below the inlet (where instability is possible) and the critical flow rate when water is used as the liquid.

One might wonder whether there are flow rates below Q^- for which further contact-line motion becomes impossible, under any circumstances. Surprisingly, one *can* identify and measure such a flow rate. Consider a newly created unsteady meander. We now slowly decrease the flow rate until all contact-line motion ceases. We call this flow rate *Q-from-above* denoted by Q^+. We find $Q^+ < Q^-$ always; higher flow rates are required to spontaneously destabilize a straight rivulet than to maintain the unsteady motion once it is begun.

We always form a straight rivulet by slowly increasing flow rate from zero. Eventually, a corrugated, highly agitated precurser to meandering is seen. At very slightly greater flow rate Q^- a beginning meander develops and evolves to a large amplitude unsteady meander. If we carefully reduce Q, we find another flow rate Q^+ at which all contact-line motion stops and the unsteady meander becomes transfixed as a steady meander. Further reduction of flow rate will cause segments of this steady meander to collapse, eventually forming a slightly less severely modulated steady meander. Still lower flow rates will destroy the entire rivulet through a capillary instability.

We have thus defined the protocol necessary for the present study.

MEASUREMENT OF CONTACT ANGLE

We measured the apparent contact angle for very slowly advancing (θ_A), and receding (θ_R) contact lines. This measurement is made with an optical goniometer which was constructed in our laboratory. The device consists of a level microscope stage upon which a specimen slide holding the test solid can be placed. The stage is backlighted by directing light through a fiber optic bundle onto the specimen through a prism assembly.

When a drop of fluid is placed onto the solid specimen slide, a semispherical sessile drop is formed. This drop can be viewed in silhouette through a microscope which focuses the outline of the drop onto a ground glass plate. By placing a reticle in the optical train, it is possible to superimpose a protractor upon the image of the drop. The contact angle is measured directly by aligning that ray of the protractor which is just tangent to the interface at the point of contact with the solid.

There is, of course, no guarantee that the contact angle associated with such a drop corresponds to either θ_A, or θ_R. To produce an advancing contact line, and so, an advancing contact angle, we introduce into the drop, a clean glass rod affixed to a dial micrometer. We move the glass rod parallel to the solid by dialling the micrometer. Capillary forces induce the contact line to move and both advancing, and receding contact lines can be produced in this way.

Repeated measures indicate that advancing and receding contact

TABLE 1. MEASURED APPARENT CONTACT ANGLES θ_A AND θ_R CORRECT TO ±3°, THE HYSTERESIS MEASURE H, AND AVERAGE VALUES OF THE INDICES I^- AND I^+

	θ_A	θ_R	H	$\bar{I}^-$	$\bar{I}^+$
HDPE	83	65	0.30	1.73	1.25
Plexiglas	74	47	0.41	1.70	1.12
Mylar	77	30	0.64	2.73	1.60
Acetate Film	65	25	0.48	2.50	1.70

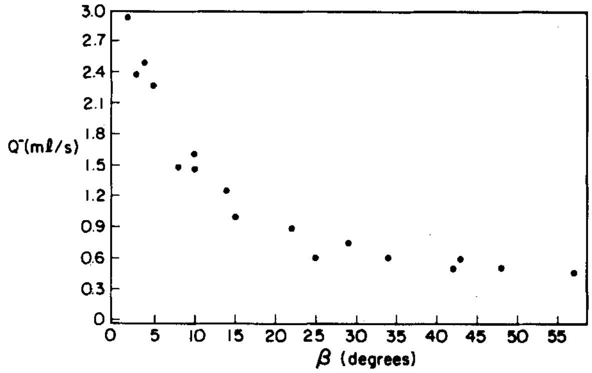


Figure 2. Q^- **(mL/s) vs.** β **(degrees) for water on HDPE.**

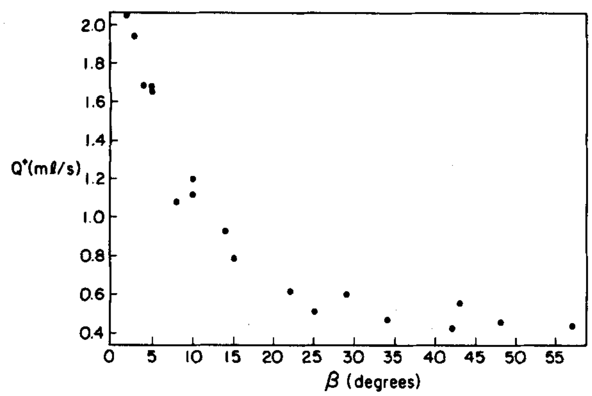


Figure 3. Q^+ **(mL/s) vs.** β **(degrees) for water on HDPE.**

angles can be measured with this device to an accuracy of ±3°. The error is partly due to error in reading the angle directly, since the line of tangency can be difficult to judge, and partly due to sample-to-sample variation. Even the same sample can produce different contact angles at different slide locations, especially when the solid surface is not smooth.

Table 1 gives the measured values of θ_A and θ_R for the material sets studied.

RIVULET EXPERIMENT

Each test plate is set in a carriage which is hinged to fix a plate-inclination angle β in the range $\beta = 0°$ (horizontal) to $\beta = 90°$ (vertical). The angle β can be measured to plus or minus 2°. A flexible tube conveys water from a reservoir to a nozzle which is held in a vise at the top of the carriage. The nozzle diameter is chosen to minimize the length of the entrance region. We use a nozzle with a three millimeter diameter. The carriage can hold plates up to 1.5 m in length, and 0.60 m in width.

The distilled water used in each run was discarded after one use. All liquid-contacting surfaces were either of glass, tygon tubing or teflon. Contamination of the solid surfaces was avoided by washing the plates with Freon TF solvent followed by a distilled water rinse before each set of runs on a given solid. Day-to-day fluctuations in temperature (20 ± 2°C) and humidity (40 to 70%) were tolerated since the size of the apparatus makes environmental control difficult. We noticed no apparent qualitative changes in system behavior due to these fluctuations and no large effects were expected.

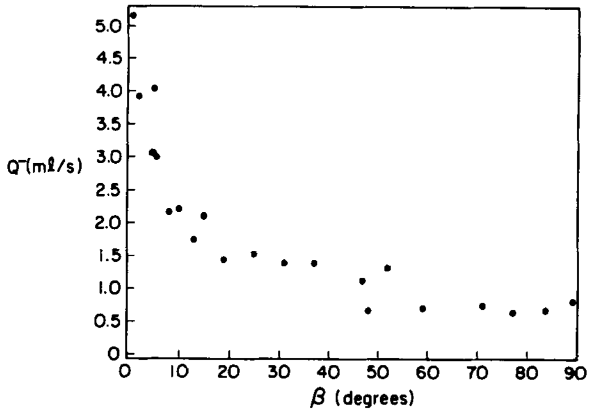


Figure 4. Q^- **(mL/s) vs.** β **(degrees) for water on Mylar.**

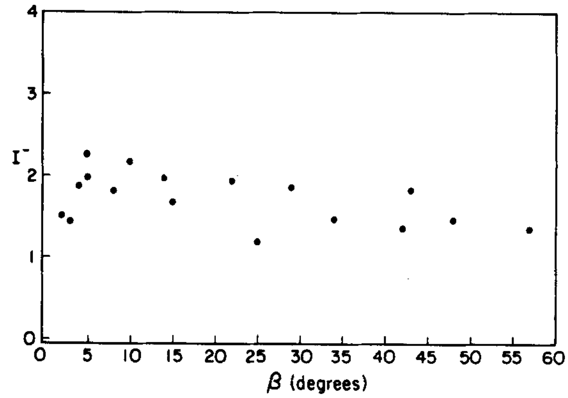


Figure 5. I^- **vs.** β **(degrees) for water on HDPE.**

The flow rate is controlled by a micrometric capillary valve which allows very fine adjustments in flow rate. The flow rate is measured with a variable cross-section flow meter to an accuracy of plus or minus 5%.

For a given plate inclination and water/substrate pair we measure Q^- and Q^+. Figures 2 and 3 show results for a number of "runs" using HDPE as the substrate. Given the use of the formation protocol, we find that Q^- and Q^+ vary systematically with plate inclination β. Note that the scatter in Q^+ is somewhat smaller than in Q^-.

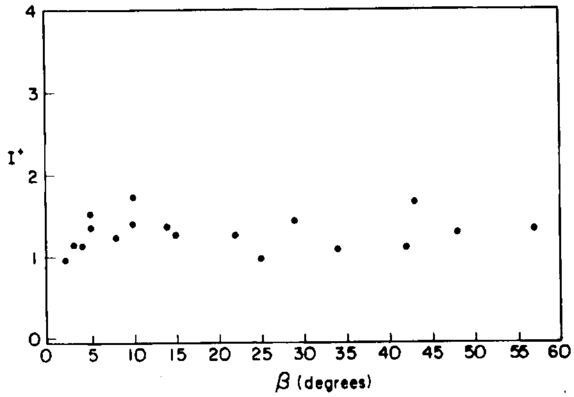
Systems with large contact-angle hysteresis require larger flow rates to produce meandering. (See Table 1 for measured θ_R and θ_A). The scatter in the data is also larger. Figure 4 shows measured Q^- values for a number of "runs" using the high hysteresis Mylar substrate. We find that for the full range of substrates considered, data for Q^- versus β and Q^+ vs. β can be well correlated using a single dimensionless index I defined by

$$I \equiv \frac{\iint_{\mathcal{A}} \rho u^2 d\mathcal{A}}{\int_{\mathcal{C}} \sigma ds} \tag{1}$$

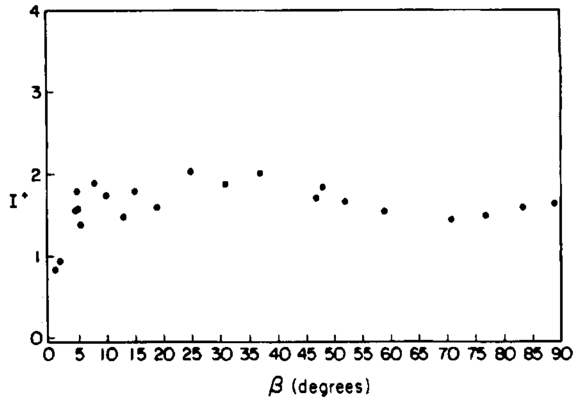
Here ρ is the liquid density, σ is the interfacial tension of the water-air interface and u is the straight-rivulet axial velocity of the liquid. For the given water flow rate Q, tilt angle β and advancing contact angle θ_A, $\mathcal{A}$ and $\mathcal{C}$ are the cross-sectional area and the arc enclosed by the cross-section, respectively, of the corresponding straight rivulet. In order to evaluate I, one must calculate numerically the velocity field and surface shape of the straight rivulet. These are defined in Towell and Rothfeld (1966).

We find that straight rivulets are unstable to meandering when I slightly exceeds unity. Consider the results for water on HDPE. The data of Q^- vs. β of Figure 2 is mapped by relation 1 into a narrow region of I^- vs. β of Figure 5 having I^- about 1.73. Similarly, the date of Q^+ vs. β of Figure 3 is mapped by relation 1 into the region of I^+ vs. β of Figure 6 having I^+ about 1.25.

The relations I vs. β for high hysteresis systems result in larger scatter. Figures 7 and 8 show data correlated through relation I for the water/Mylar



Figuer 6. I^+ vs. β (degrees) for water on HDPE.



Figuer 8. I^+ vs. β (degrees) for water on Mylar.

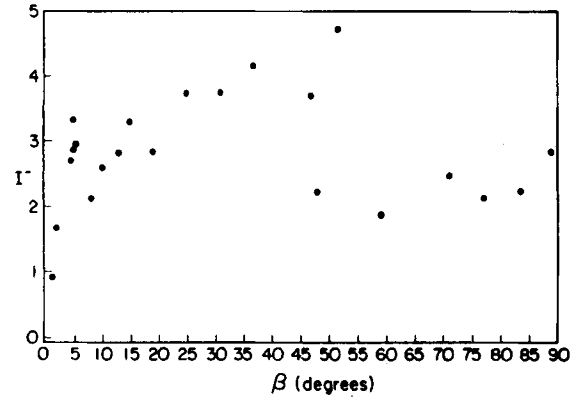


Figure 7. I^- vs. β (degrees) for water on Mylar.

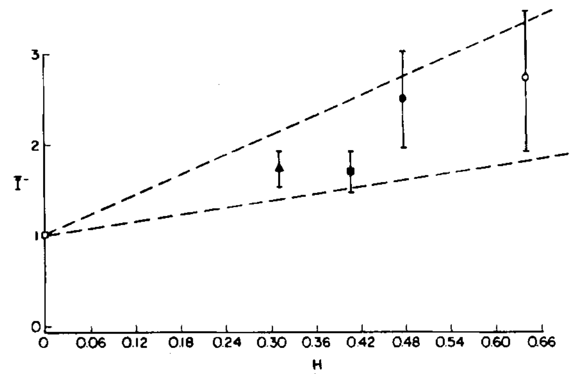


Figure 9. $\bar{I}^-$ vs. H: □ "Free," ▲ Mylar, ■ Plexiglas, ● Acetate, ○ HDPE.

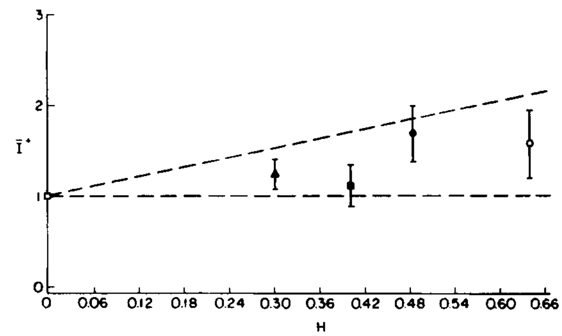


Figure 10. $\bar{I}^+$ vs. H: □ "Free," ▲ Mylar, ■ Plexiglas, ● Acetate, ○ HDPE.

system. Raw data for all the above-mentioned substrates as well as the corresponding correlations through relation 1 are given in Culkin (1981).

The formulation of index I involves regarding the straight rivulet as slender and considering a gentle virtual displacement of the rivulet corresponding to a nascent meander. If the rivulet Reynolds number is large, then one balances the destabilizing pressure gradient created by curving the streamlines with the stabilizing interfacial forces tending to minimize the interfacial area of the rivulet. These two effects are respectively proportional to the numerator and denominator of I. If only these two effects limit the instability, then $I < 1$ would be the required stability criterion. However, contact-angle hysteresis effects at the contact lines will retard the instability so that the critical value of I will exceed unity. More details of the arguments and calculations can be found in Culkin (1981). Since I is essentially independent of β, we can compare the stability criteria I^- and I^+ for different substrates by using average values $\bar{I}^-$ and $\bar{I}^+$, the averages taken over all "runs" for the given substrate. These are given in Table 1.

We can measure contact-angle hysteresis by an index H,

$$H = \cos\theta_R - \cos\theta_A. \tag{2}$$

given in Table 1. Thus, we relate $\bar{I}^-$ and $\bar{I}^+$ to H in Figures 9 and 10 for all four substrates considered. Here we require that all "theoretical" curves pass through the point $H = 0, \bar{I} = 1$. The error bars in Figures 9 and 10 encompass about 80% of all data in order to stress the fact that increased contact-angle hysteresis produces increased uncertainty about the values of $\bar{I}^{\pm}$. In the same way, the dashed lines in Figures 9 and 10 are simply estimates which serve, for engineering purposes, to separate the plane into regions where contact-line motion is likely (above the upper dashed line), uncertain (between lines) and unlikely (below the lower dashed line). Within the limitations imposed by the relatively large scatter associated with the large hysteresis systems, we note that spontaneous transition to meandering occurs when $\bar{I}$ lies above the upper dashed lines of Figure 9. When $\bar{I}$ lies below the lower dashed curve of Figure 9, then the straight rivulet persists. In order to sustain an unsteady meander as Q is decreased, $\bar{I}$ must remain above the upper dashed line of Figure 10. If $\bar{I}$ is below the dashed line of Figure 10, then the unsteadiness of the meander ceases and either a stationary meander or the straight rivulet results. Reading left to right, the points correspond to HDPE, Plexiglas, Acetate film, and Mylar.

The stability index I is sensitive to changes in Q since a 1% change in Q results in about a 1% change in I. Hence, an accurate estimate of I leads to an accurate estimate of the critical conditions needed to produce or eliminate meanders.

CONCLUSIONS

The stability index I successfully predicts transition to meandering from the straight rivulet and subsequent cessation of contact-line motion in the unsteady meander. Unfortunately, the index is useful only when the Reynolds number, based on flow rate and cross-sectional dimensions, of the corresponding straight rivulet is high (≈2,000). Rivulets can meander at values of I far less than unity when the liquid viscosity is high and the corresponding Reynolds number is low, apparently through a mechanism entirely different from that envisioned here for water rivulets. Culkin (1981) finds that rivulets of water-glycerine mixtures meander soon after large amplitude travelling waves appear on the free surface. The resulting motion of the contact lines is described and quantitative data for the critical flow rate to produce meandering are presented.

It is interesting to note that the high Reynolds number analysis of the stability of water rivulets succeeds because of the presence of significant contact angle hysteresis. The hysteresis force stabilizes the position of the contact lines and prevents the usual linear instability (Davis, 1980). Only when the hydrodynamic disturbances grow to sizes sufficient to overcome hysteresis can the contact lines move, and this seems to occur at high Reynolds number in the case of water rivulets.

We have shown that under carefully controlled conditions leading to rivulet formation, the critical flow rate at which spontaneous meandering begins (Q-from-below) and the critical flow rate needed to sustain unsteady meandering (Q-from-above) are reproducible and well defined. These useful concepts lead to a simple correlation between the behavior of water rivulets upon a variety of materials and a single dimensionless number, the stability parameter I. For many engineering applications, this correlation provides a simple means by which the gross flow characteristics of water rivulets can be estimated, given the liquid materials properties, and advancing and receding contact angles, and the inclination of the solid. In systems with large contact-angle hysteresis, much larger flow rates are required to produce and sustain unsteady meandering, and the error of estimation for the exact transition point becomes large.

ACKNOWLEDGMENT

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NOTATION

$\mathcal{A}$ = surface area
$\mathcal{C}$ = surface contour
H = hysteresis index defined in Eq. 2
I = stability index defined in Eq. 1
$\bar{I}$ = average value of I
Q = flow rate
s = arc length
u = axial velocity in straight rivulet

Greek Letters

β = inclination angle of plate
ρ = density of liquid
σ = interfacial tension
θ = contact angle
θ_A = advancing contact angle
θ_R = receding contact angle

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Experimental Study of Deep Bed Filtration: A Stochastic Treatment

A stochastic pure birth process, which describes pore blockage in a filtration process, has been coupled with the Carman-Kozeny equation to simulate the pressure buildup in deep bed filtration. It has been shown that the resultant one parameter model equation fits adequately the experimental data obtained under the straining dominated condition, and that available data obtained under the adhesion dominated condition can also be described by the model.

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SCOPE

Granular filters have long been used to remove suspended solids from water in both the purification of potable water and the treatment of waste water (Fuller, 1898; Dunbar and Calvert, 1908; Hall, 1957). The suspended solids are retained in the bed either through a straining mechanism or an adhesion mechanism (see, e.g., Tien and Payatakes, 1979). The straining mechanism dominates when the characteristic diameter of the suspended solids is large relative to the pore diameter in the bed matrix. Conversely, the adhesion mechanism dominates when the characteristic diameter is much smaller than that of the pore. Adhesion of the suspended solids to the collector particles takes

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